Synopsis of the sixth lecture in ECO 5315 (Game Theory)

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Introduction

Game theory is a method for studying strategic situations. Much of the initial funding for game theory research came from the War Department during World War II, and continued after the war under the auspices of the RAND Corporation.[1] Early on, applications were developed for determining war strategies and for modeling the dynamics of arms races, but researchers also found a number of useful real world applications in social science disciplines such as economics, political science, and sociology.[2]

Of course, our interest is in the economics applications of the game theory. Before describing strategic situations, it is useful to think of examples that are *not* strategic in nature. Two obvious examples come to mind; specifically, perfect competition (since all firms are price takers, they don't have to worry about actions of competitors) and monopoly (since there is no competition, so there is no need to worry about actions of competitors, because there are none!). However, oligopoly represents a strategic situation, since with a small number of firms, your outcomes are affected not only by your own actions, but also by actions of your competitors.

Thus, game theory is relevant and applicable whenever the consequence of a manager's decision depends on both the manager's own action and the actions of others. Furthermore, there are no unconditional "optimal" strategies in game theory; the optimality of a strategy depends on the situation in which it is implemented.

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[1]After World War II (in 1947), the War Department was renamed the Department of Defense). The RAND Corporation was an outgrowth of Project RAND (RAND being an acronym for "Research and Development").

[2]See http://en.wikipedia.org/wiki/Game_theory#History for an interesting historical account of the development of the game theory. As this article points out, elements of game theory were "in play" (pardon my pun) going as far back as the early 18th century, and as this article points out, "James Madison made (in an article published in 1787) what we now recognize as a game-theoretic analysis of the ways states can be expected to behave under different systems of taxation". As early as 1838, Cournot developed a model of duopoly (which we covered in Chapter 10) that anticipates by more than a century the famous Nash equilibrium concept. The foundations for using game theory in economics were introduced in a seminal book published in 1944 by John von Neumann and Oskar Morgenstern entitled *Theory of Games and Economic Behavior*.

Game Theory Requirements

In order to have a game, one needs players (e.g., firm 1 and firm 2), strategies (e.g., increase price or maintain price), outcomes or consequences (e.g., succeed or fail), and payoffs (e.g., succeed pays off $5; fail imposes a $5 loss). Also, we need to know the order of play; i.e., do players play simultaneously or sequentially?

We are interested in determining whether games have so-called "equilibrium" outcomes. In equilibrium, no player has the incentive to unilaterally change her strategy; conditional on the choices of others, she is doing the best she can. Equilibria are *rational*, *optimal*, and *stable* in the following ways:

- Equilibria are *rational* in the sense that players are motivated to seek the best possible outcomes for themselves;

- Equilibria are *optimal* because, conditional on the choices of others, players cannot do any better than the equilibrium outcome; and

- Equilibria are *stable*; unless player 2's behavior changes, there is no reason for player 1 to change her behavior.

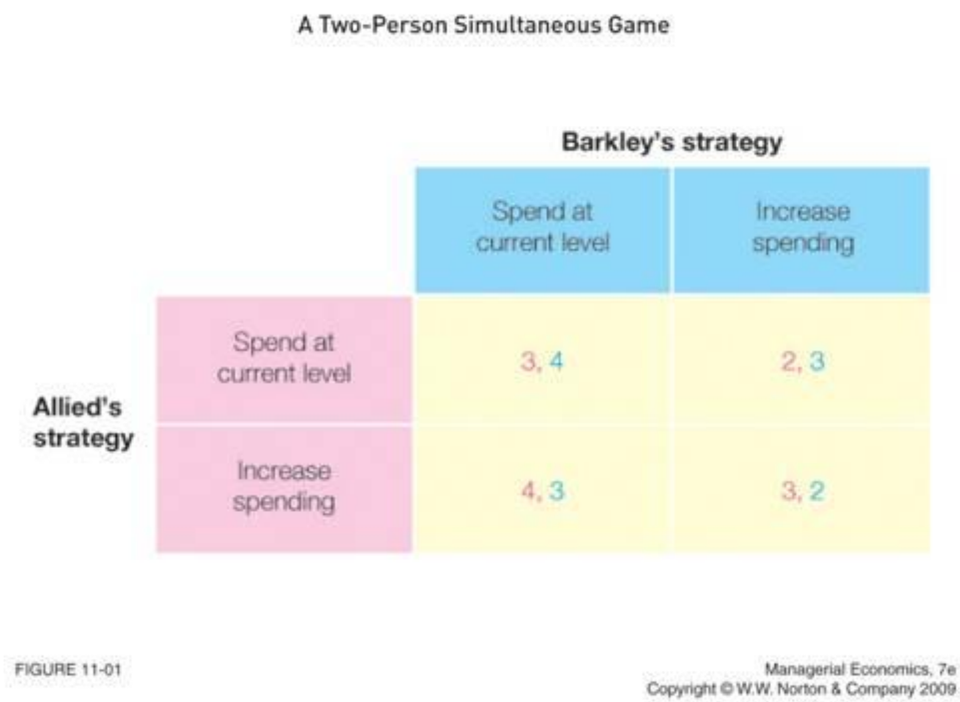
Dominant Strategies

Game-theoretic predictions and advice about how to play a game rest on a collection of criteria known as "solution concepts." The first solution concept that we considered during the game theory lecture was that of a *dominant strategy*. There are two forms of dominant strategies; strong and weak:

- <u>Strong Dominance</u>: A strategy whose payout in any outcome is higher relative to all other feasible strategies is *strongly dominant*.

- <u>Weak Dominance</u>: A strategy is *weakly* dominant if it does at least as well as any other strategy for some outcomes (i.e., it's tied with another for the highest payoff) and better than any strategy for the remaining outcomes.

Figure 11-1 in the textbook provides a very clear example of a situation in which both players (in this case, firms called Allied and Barkley) have *strongly dominant strategies*. Allied is the "row" player (pink), and Barkley is the "column" player (blue). The ordering of the payoffs matters – the convention in a matrix is that the first number is the payoff for the row player, and the second is the payoff for the column player. A strictly dominant strategy exists for a player when one strategy is always better than another strategy, irrespective of what the opponent does. In this example, for Allied, "Increase spending" strictly dominates "Spend at current level", since "Increase spending" pays off 4 when Barkley chooses "Spend at current level" and 3 when Barkley chooses "Increase spending", compared with payoffs of 3 and 2 from Allied playing "Spend at current level in the same states. Similarly, for Barkley, "Spend at current level" strictly dominates "Increase spending", since "Spend at current level" pays off 4 when Allied chooses "Spend at current level" and 3 when Allied chooses "Increase spending", compared with payoffs of 2 and 3 from Barkley playing "Increase spending" in the same states.


A Two-Person Simultaneous Game

		Barkley's strategy	
		Spend at current level	Increase spending
Allied's strategy	Spend at current level	3, 4	2, 3
	Increase spending	4, 3	3, 2

FIGURE 11-01

The "Split or Steal" game that we discussed in class provides an excellent example of *weak dominance*. The payoff matrix for "Split or Steal" was given as follows:

		Player 2	
		Steal	**Split**
Player 1	**Steal**	0,0	100,0
	Split	0,100	50,50

The payoffs indicated in the "Split or Steal" game indicate the proportion of the jackpot that each player receives. There is no *strong* dominance here; however, there is *weak* dominance. To see this, let's think through the strategy choices of the two players.

Suppose Player 2 plays Steal. In this case, the payoffs for Player 1 are 0 if she Steals and 0 if she Splits. Now suppose Player 2 plays Split. In this case, payoffs for Player 1 are 100 if she Steals and 50 if she Splits. Thus Steal weakly dominates Split for Player 1, since it *ties* with Split when Player 2 plays Steal, and it is *better* than Split when Player 1 plays split. By symmetry, Steal weakly dominates Split for Player 2 as well.

Nash Equilibrium

How can managers anticipate behavior in games in which a dominant strategy equilibrium does not exist? John Nash's answer to this question is the most widely used solution concept in game theory: the Nash equilibrium. Assuming that players are rational, every player chooses the "best" strategy conditional on all other players doing the same.

To see this, consider the new product introduction problem shown in Figure 11.6, p. 382:

		Allied		
		Product alpha	Product beta	Product zeta
	Product lambda	4, 6	9, 8	6, 10
Barkley	Product pi	6, 8	8, 9	7, 8
	Product sigma	9, 8	7, 7	5, 5

Let's check to see whether there is a dominant strategy:

- Barkley prefers lamda over pi and sigma when Allied plays beta;
- Barkley prefers pi over lambda and sigma when Allied plays zeta;
- Barkley prefers sigma over pi or lambda when Allied plays alpha.
- Therefore, there is no dominant strategy equilibrium – what to do???

Economics Nobel Laureate John Nash's answer to this question is the most widely used solution concept in game theory: the Nash equilibrium. Assuming that players are rational, every player chooses the "best" strategy conditional on all other players doing the same. Therefore, it is useful to think of the Nash equilibrium as a *conditional dominant strategy*. The key here is to put yourself into other people's shoes; i.e.,

- If you were a Barkley manager and knew that Allied would introduce alpha, what would you do? Play sigma!
- If you were a Barkley manager and knew that Allied would introduce beta, what would you do? Play lambda!
- If you were a Barkley manager and knew that that Allied would introduce zeta, what would you do? Play pi!
- If you were an Allied manager and knew that Barkley would introduce lambda, what would you do? Play zeta!
- If you were an Allied manager and knew that Barkley would introduce pi, what would you do? Play beta!
- If you were an Allied manager and knew that Barkley would introduce sigma, what would you do? Play alpha!

Thus the Nash equilibrium involves Barkley introducing product sigma and Allied introducing product alpha.

Prisoners Dilemma

Sometimes, rational play by rational players can lead to bad outcomes. When this happens, we have what is commonly referred to as a Prisoners Dilemma. A Prisoners Dilemma occurs when the players of a game rationally choose not to cooperate with each other even if it is in their best interests to do so. This is clearly illustrated in the Split or Steal game referenced earlier. There, we found that "Steal" weakly dominates "Split" for both players. It is a Prisoners Dilemma in the sense that if hypothetically given the choice between the Split, Split (50,50) and the Steal, Steal (0,0) payoffs, both players would prefer Split, Split. Unfortunately, there is no way for them to arrive at this state because of the incompatibility of incentives that is part and parcel of the game's structure.

Prisoners Dilemmas occur quite frequently in the real world. I gave some examples in class, including the use of steroids in athletic competition, bank runs, piracy of digital media, and pollution (see http://bit.ly/oQ7qt for even more examples).

Remedies often include the imposition of various forms of regulatory, legal, and contractual innovations, etc. The goal of such remedies is to change payoffs such that Nash equilibria result in more favorable outcomes. As always, there are tradeoffs associated with policy responses. For example, in the case of bank runs, deposit insurance solves the immediate problem; i.e., there's no need for a run on the bank if your deposit is insured. However, it creates a potentially larger (moral hazard) problem, in the sense that it removes (or at the very least, substantially mitigates) an important dimension of market discipline; specifically, monitoring of bank solvency by depositors. Thus bankers no longer have to compete for depositors on the dimension of risk, which in turn increases the likelihood that banks may take on excessive risk. We'll study the moral hazard problem in more detail during class on Monday, November 30.

Strategic Foresight and Backward Induction

The last topic that we covered was the use of backward induction for the purpose of enhancing one's ability to make decisions today that are rational given what is anticipated in the future. Backward induction involves looking to the future, anticipating what strategies rational players will choose, and then deciding upon an appropriate course of action today based upon these beliefs. In sequential games, backward induction involves starting with the last decisions in the sequence and then working backward to the first decisions, identifying all optimal decisions along the way.