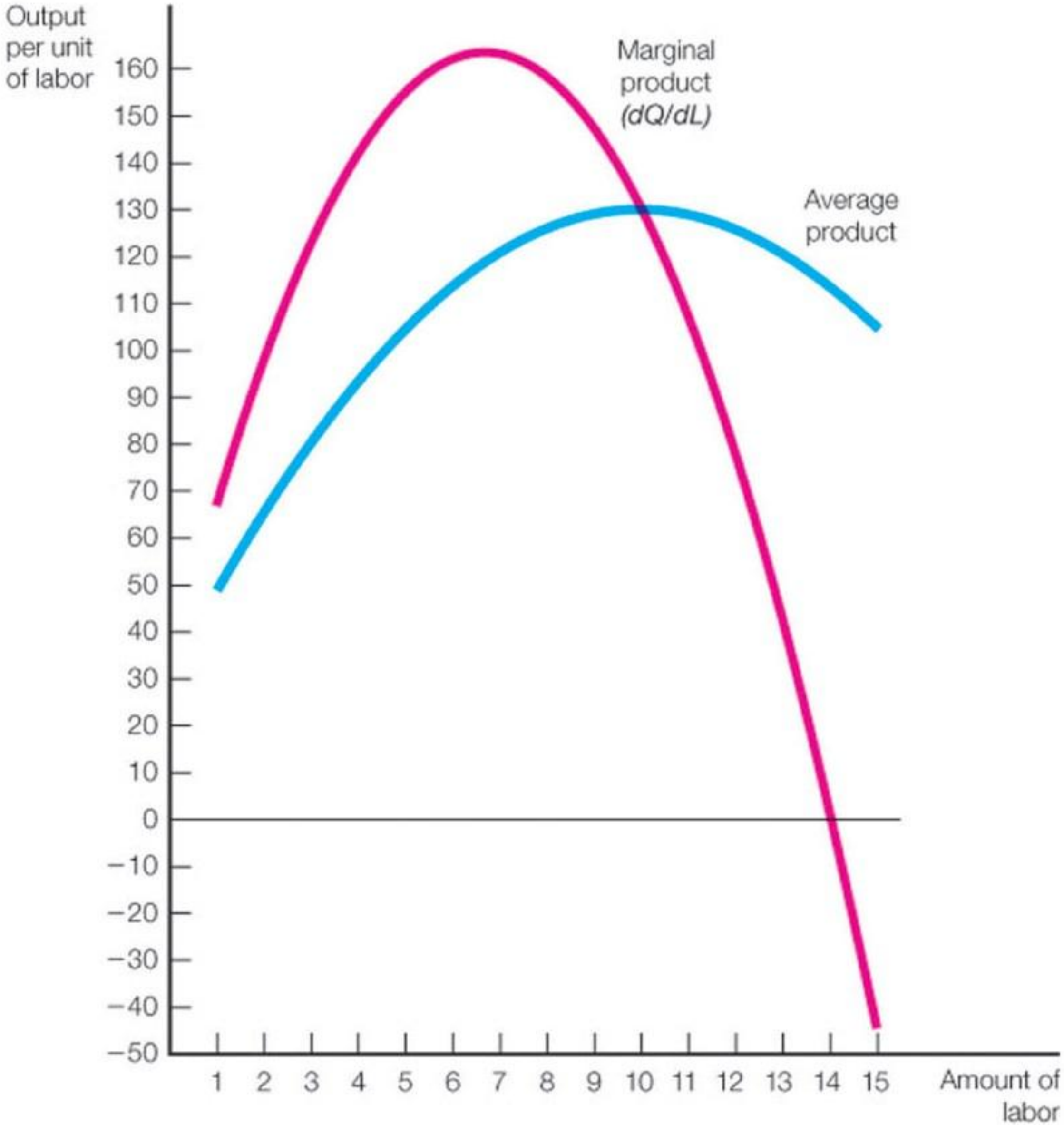
Lecture 3

Production Theory

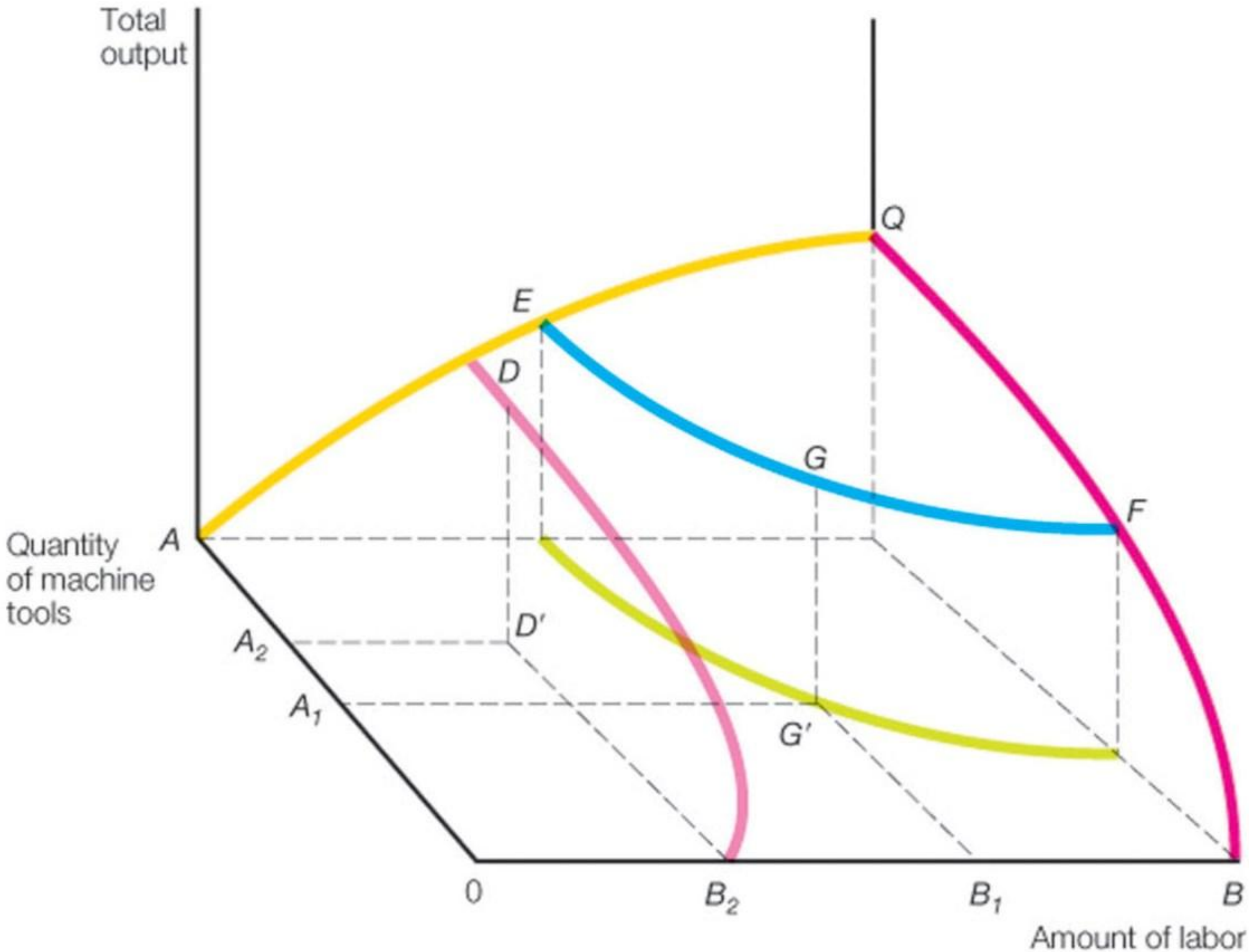
Production Theory (Continued)

- Last time, we introduced the notion of a *production function*, which mathematically characterizes the relationships between inputs and output for firm; typically, we assume that output (Q) is a function of capital and labor inputs; i.e., $Q = f(L, K)$.
- We also introduced the concepts of marginal ($MP_L = dQ/dL$) and average product of labor ($AP_L = Q/L$) (holding K constant).
  - We showed (among other things) that $MP_L = AP_L$ when AP_L is maximized!

Average and Marginal Product Curves for Labor



Production Function, Two Variable Inputs



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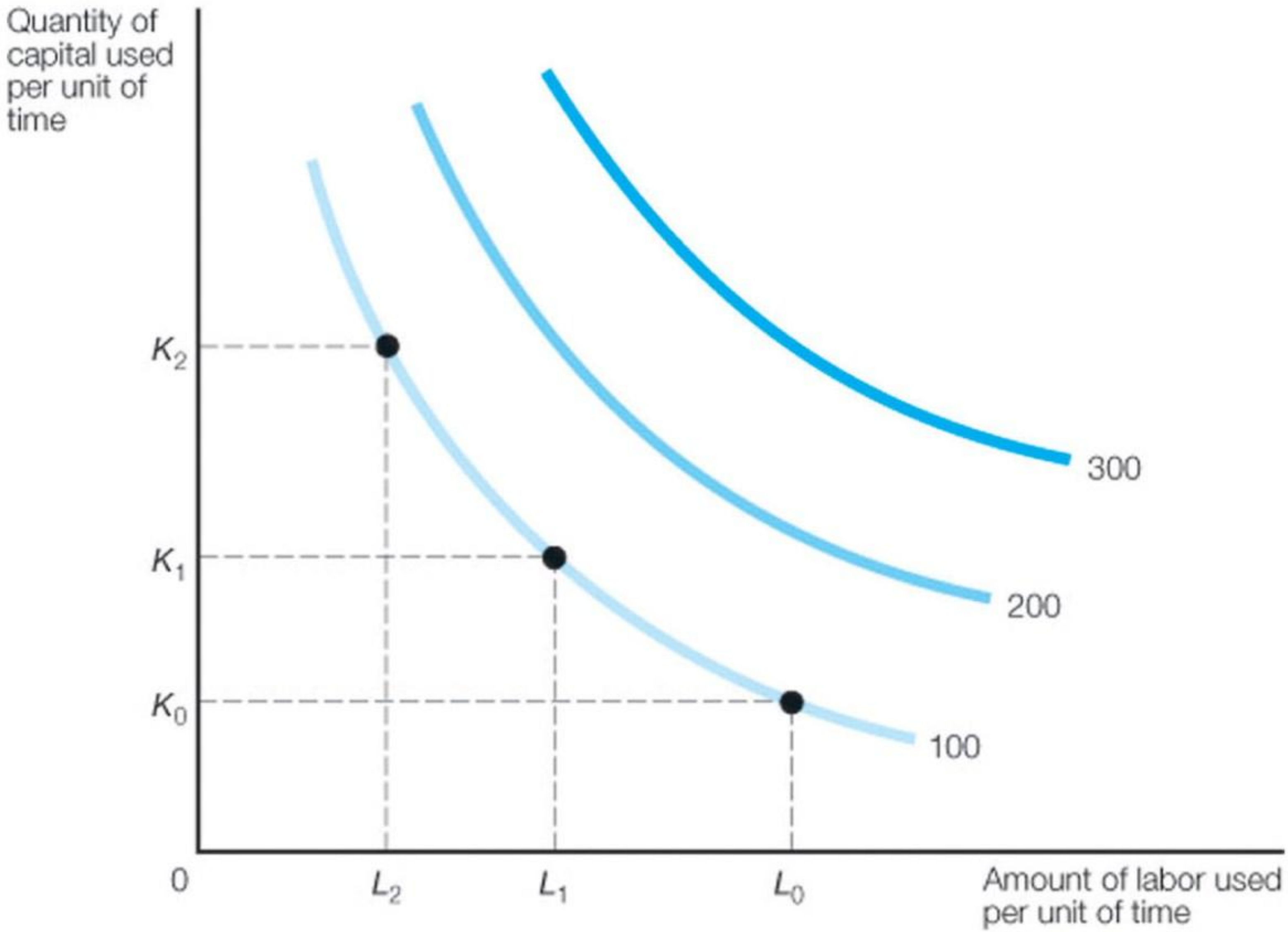
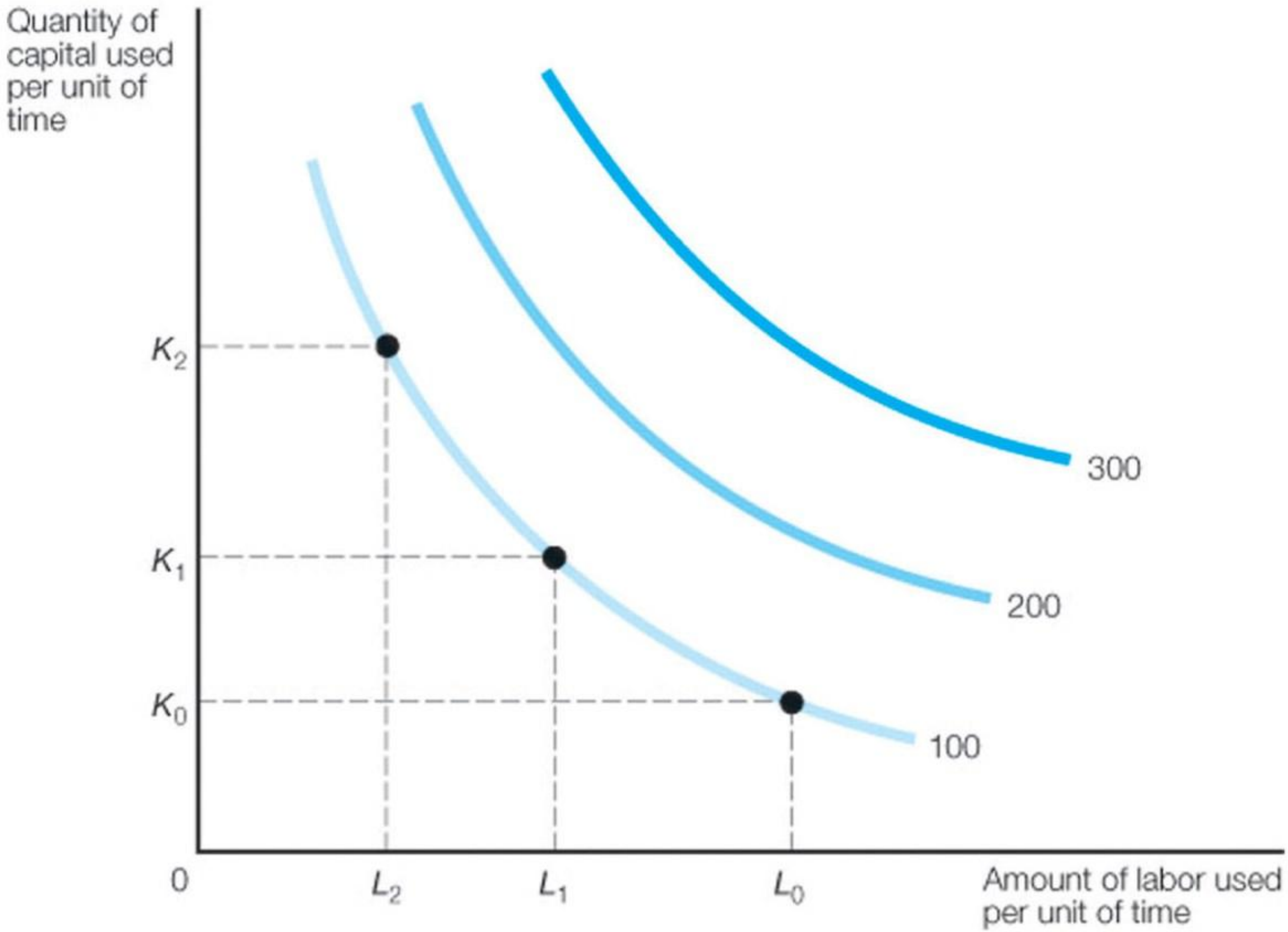


Quantity of capital used per unit of time
K_2
K_1
K_0
300
200
100
0
L_2
L_1
L_0
Amount of labor used per unit of time

Marginal Rate of Technical Substitution

- The marginal rate of technical substitution (*MRTS*) indicates the rate at which one input must be substituted for another in order for output to remain constant.
- Suppose output is a function of the amounts of two inputs; i.e., $Q = f(L, K)$. Then $MRTS = -\frac{dK}{dL}$, or -1 times the slope of the isoquant.
- We can also express MRTS as the ratio of marginal products; i.e., $MRTS = \frac{MP_L}{MP_K}$.

Isoquants


Quantity of capital used per unit of time
K2
K1
K0
300
200
100
0
L2
L1
L0
Amount of labor used per unit of time

The Optimal Combination of Inputs

What combination of capital and labor should the firm choose so as to maximize the quantity of output for a given cost?

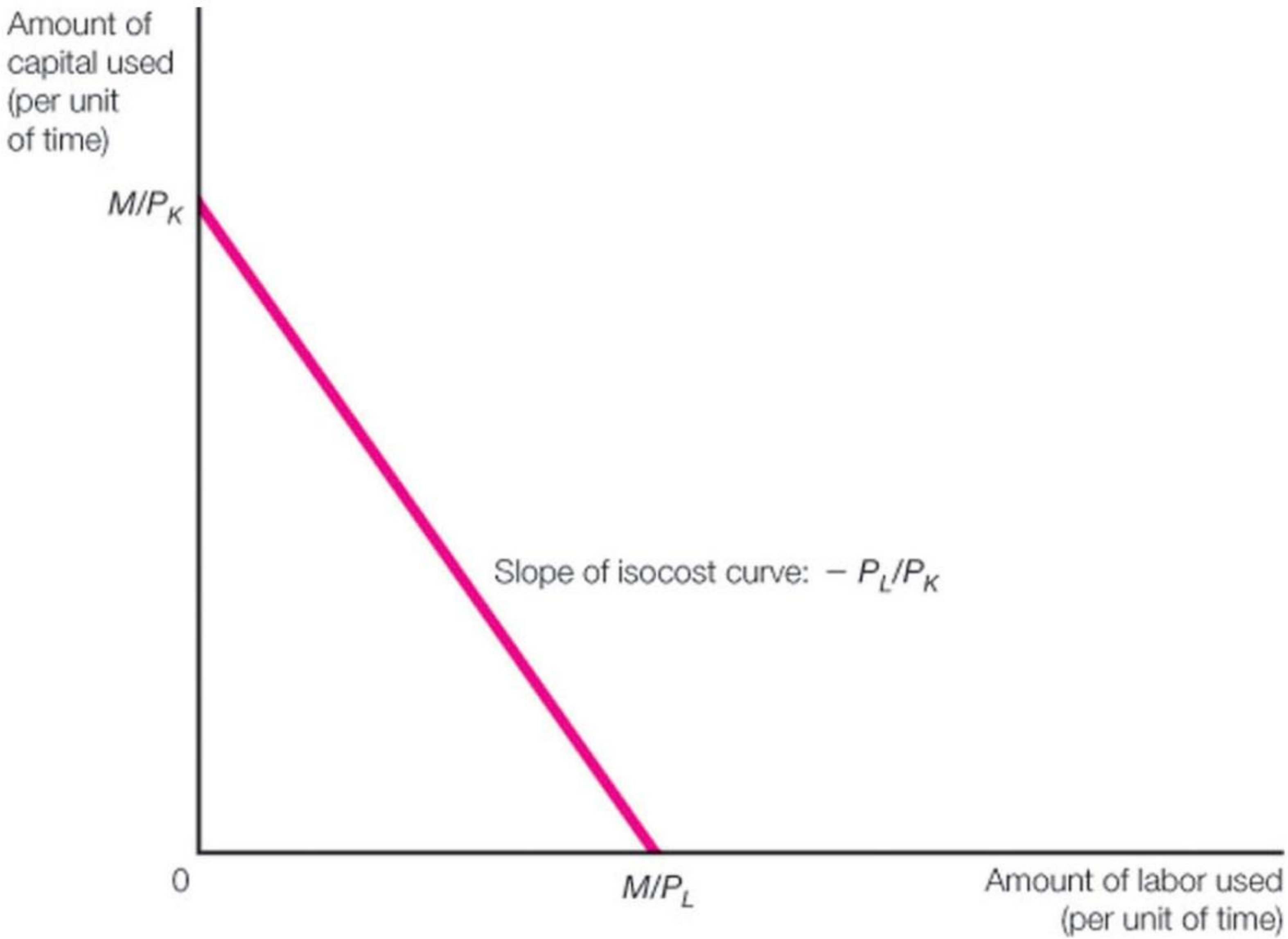
$$M = P_L L + P_K K,$$

where M is the amount paid for inputs, the P's are the unit prices of inputs, and L and K are the number of labor and capital units.

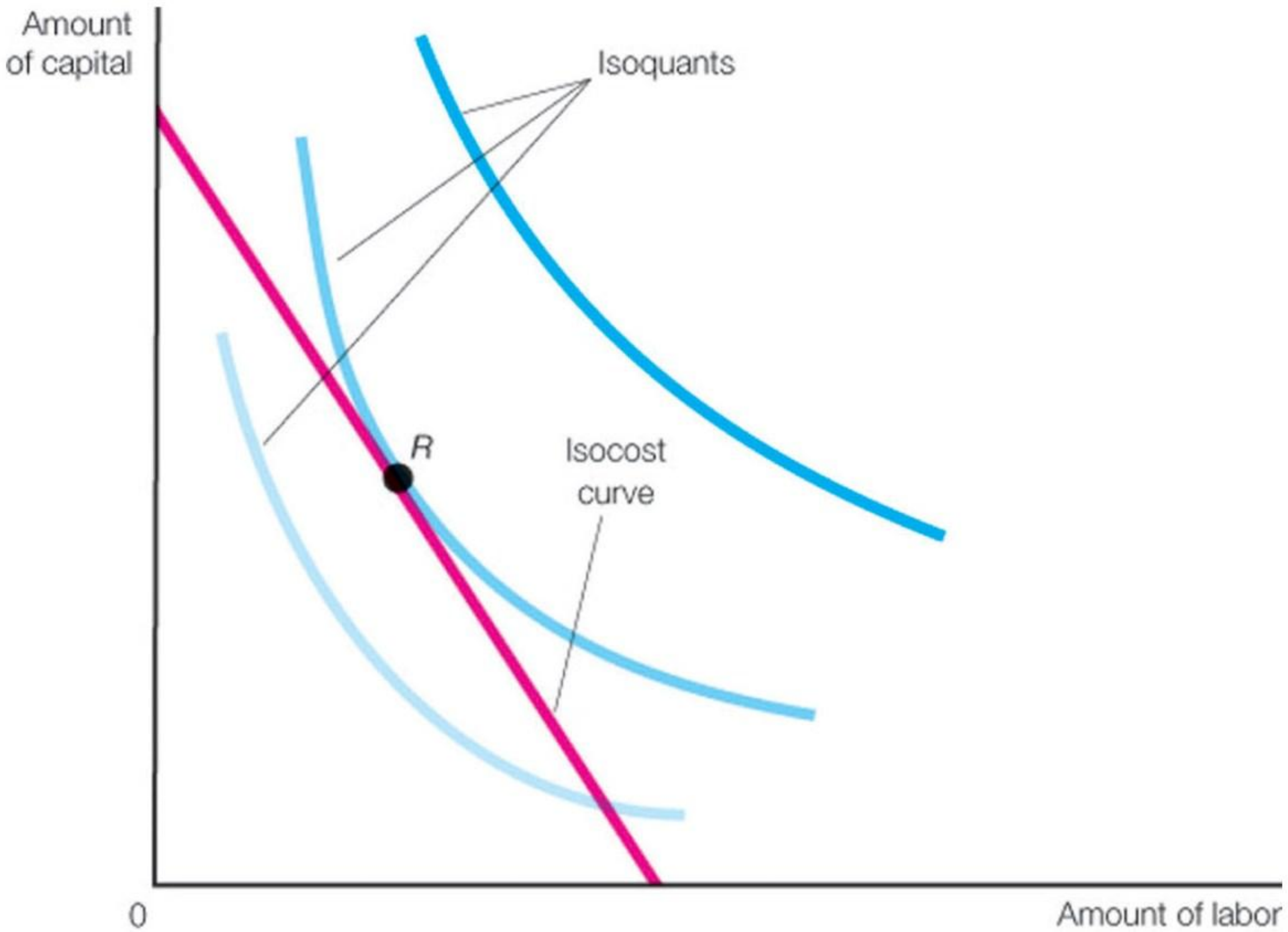
The isocost curve (actually, line) is found by solving for K:

$$K = \frac{M}{P_K} - \frac{P_L}{P_K} L.$$

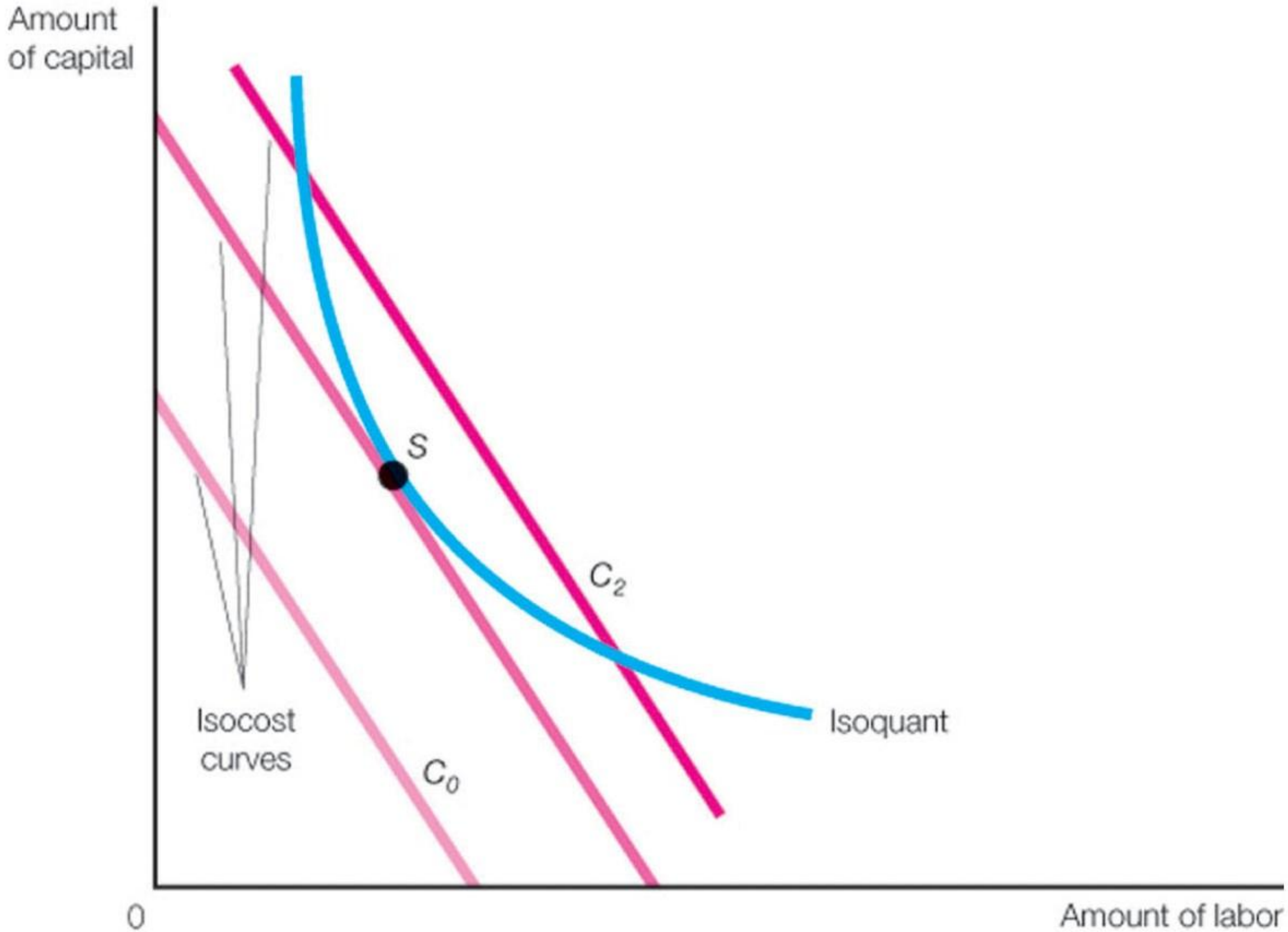
Isocost Curve



Maximization of Output for a Given Cost



Minimization of Cost for a Given Output



Class Problem 4.1

- Consider the Beiswanger Company, a small firm engaged in engineering analysis. Beiswanger's president has estimated that the firm's output per month (Q) is related in the following way to the number of engineers (E) and technicians used (T): $Q = 20E - E^2 + 12T - 0.5T^2$.

The monthly wage of an engineer is $4,000. and the monthly wage of a technician is $2,000. If the president allots $28,000 per month for the combined wages of engineers and technicians, what mix of engineers and technicians should he hire?

Class Problem 4.2

- Managers need to search for input bundles to minimize costs for a given output. Intuitively they need to balance the productivity of an input with its cost. Consider the Miller Company, for which the relationship between output per hour (*Q*) and the number of workers (*L*) and machines (*K*) used per hour is $Q = 10(LK)^{0.5}$.

The wage of a worker is $80 per hour, and the price of a machine is $20 per hour. If the Miller Company produces 800 units of output per hour, how many workers and machines should managers use?

Class Problem 4.3

- Happiness can be produced with wine and roses according to $Q = W^{1/2}R^{1/4}$, where W is bottles of wine and R is bouquets of roses obtained per month. If wine costs \$20 per bottle and roses cost \$60 per dozen, find the happiness maximizing combination of wine and roses costing \$360.

Output Elasticity and Returns to Scale

- Suppose $Q = aL^b K^c$. Next, we compute the output elasticities for labor (η_L) and capital (η_K):

$$\eta_L = \frac{\partial Q}{\partial L}\frac{L}{Q}. \quad \frac{\partial Q}{\partial L} = abL^{b-1}K^c, \text{ so } \eta_L = \frac{abL^b K^c}{aL^b K^c} = b.$$

$$\eta_K = \frac{\partial Q}{\partial K}\frac{K}{Q}. \quad \frac{\partial Q}{\partial K} = acL^b K^{c-1}, \text{ so } \eta_K = \frac{acL^b K^c}{aL^b K^c} = c.$$

- Constant, decreasing, and increasing returns to scale:
  - If $b + c = 1$, returns to scale are *constant*;
  - If $b + c < 1$, returns to scale are *decreasing*; and
  - If $b + c > 1$, returns to scale are *increasing*.