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DEPARTMENT OF ECONOMICS

ECO 5315 Chapter 14 and Chapter 15 Problem Set Solutions

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1. (Problem 2, p. 502 in the textbook) Suppose the typical Florida resident has wealth of \$500,000, of which his or her home is worth \$100,000. Unfortunately, Florida is in hurricane alley, and it is believed there is a 10 percent chance of a hurricane that could totally destroy the house (i.e., a loss of \$100,000). However, it is possible to retrofit the house with various protective devices (shutters, roof bolts, etc.) for a cost of \$2,000. This reduces the size of loss from a 10 percent chance of loss of \$100,000 to a 5 percent chance of a loss of \$50,000. The homeowner must decide whether to retrofit and thereby reduce the expected loss. The problem for an insurance company is that it does not know whether the retrofit will be chosen and therefore cannot quote a premium conditioned on the policyholder choosing this action. Nevertheless, the insurance company offers the following two policies from which the homeowner can choose: (1) The premium for insurance covering total loss is \$12,000 or (2) the premium for insurance covering only 50 percent of loss is \$1,500. The typical homeowner has a utility function equal to the square root of wealth. Will the homeowner retrofit the house, and which insurance policy will the homeowner buy? Will the insurance company make a profit (on average) given the homeowner's choice?

Scenario #1: Full insurance coverage and no retrofit:

$$EW = (500,000 - 12,000 - 2,000) = \$486,000,$$

$$EU = (500,000 - 12,000)^{0.5} = 698.57, \text{ and}$$

$$E\pi = 12,000 - (0.1)(100,000) = \$2,000.$$

Scenario #2: Full insurance coverage and retrofit:

$$EW = (500,000 - 12,000 - 2,000) = \$486,000,$$

$$EU = (500,000 - 12,000 - 2,000)^{0.5} = 697.14, \text{ and}$$

$$E\pi = 12,000 - (0.05)(50,000) = \$9,500.$$

Scenario #3: Partial insurance coverage and no retrofit:

$$EW = .1(500,000 - 1,500 - (1-.5)100,000) + .9(500,000 - 1,500) = \$493,500,$$

$$EU = .1(500,000 - 1,500 - (1-.5)100,000)^{0.5} + .9(500,000 - 1,500)^{0.5} = 702.4109,$$

$$E\pi = 1,500 - (0.10)(.5)(100,000) = -\$3,500.$$

Scenario #4: Partial insurance coverage and retrofit:

$$EW = .05(500,000 - 1,500 - (1-.5)50,000 - \$2,000) + .95(500,000 - 1,500 - \$2,000) = \$495,250,$$

$$EU = .05(500,000 - 1,500 - (1-.5)50,000 - 2,000)^{0.5} + .95(500,000 - 1,500 - 2,000)^{0.5} = 703.7291,$$

$$E\pi = 1,500 - (0.05)(.5)(50,000) = \$250.$$

CONCLUSION: The home owner will choose to retrofit and buy the 50 percent insurance coverage because this offers her the highest expected utility. Given this choice by the home owner, the insurance company will make an expected profit of \$250.

Note: This analysis implicitly assumes that insurance is compulsory¹. If insurance is not compulsory, then the homeowner should forego insurance and retrofit, since this will provide the highest expected utility:

Scenario #5: No insurance coverage and retrofit:

$$EW = .05(500,000 - 50,000 - 2,000) + .95(500,000 - 1,500 - 2,000) = \$495,500,$$

$$EU = .05(500,000 - 50,000 - 2,000)^{0.5} + .95(500,000 - 1,500 - 2,000)^{0.5} = 703.873,$$

Scenario #6: No insurance coverage and no retrofit:

$$EW = .1(500,000 - 1000,000) + .9(500,000) = \$490,000,$$

$$EU = .1(500,000 - 1000,000)^{0.5} + .9(500,000)^{0.5} = 699.642.$$

- (Problem 4, page 503 in the textbook) A firm used to have productive assets that generated an income stream with a present value (PV) of 3,000. However, fire occurred and most of those assets were destroyed. The remaining, undamaged assets produce an income stream that has a present value of only 1,000. Therefore, the fire has led to reduction in the value of the firm from 3,000 to 1,000. The firm could undertake a reconstruction of the damaged assets for a capital cost of 1,500, which would restore the income stream to its preloss level ($PV = 3,000$). The firm has existing debt of 2,000, which is a senior claim. Would the shareholders choose to reinvest by issuing new equity to pay for the loss or are they better off walking away from the firm? Would the decision made by the shareholders be in the best interests of the bondholders? In answering this question remember that the shareholders have limited liability and therefore the share value cannot be negative.

SOLUTION: The decisions and payouts can be represented as shown in the following table. Note the total gain in value for the firm is \$2,000. If the firm does not reinvest, then total value is \$1,000 but \$2,000 is owed in debt. The firm is bankrupt, so the shareholders will use limited liability to walk away and equity will be worth zero. In this case, the firm defaults on the debt and it will be worth only \$1,000 (there is simply not enough value in the firm to pay the whole \$2,000 owing to the creditors). If the shareholders decide to reinvest, they must raise \$1,500. This seems like a good deal to the firm as a whole since the investment of \$1,500 raises the total value by \$2,000 (from \$1,000 to \$3,000). But where does this gain go? First, the creditors can now be paid in full (they are senior and have priority over shareholders). This leaves a gain of only \$1,000 for the shareholders. From the shareholders' perspective, they have paid \$1,500 to

¹It turns out that if insurance is not compulsory, home owners will prefer to self insure and mitigate. As a practical matter, one cannot self-insure a property risk like this unless one has a 100% equity position in the asset. Since the mortgage holder has an insurable interest in the value of the property which collateralizes the mortgage debt, property insurance is generally required (as per a covenant listed in the mortgage loan documents), so it seems reasonable to assume that the insurance is effectively compulsory.

raise the value of their equity by only \$1,000 (from zero to \$1,000), yielding a net loss of \$500. The shareholders would not choose to reinvest. This leaves the creditors in a default situation, getting only 50 cents payback on each dollar of debt.

Value of firm, debt, and equity with and without reinvestment			
Decision	Firm	Debt	Equity
Reinvest	\$3,000	\$2,000	\$1,000
Not reinvest	\$1,000	\$1,000	\$0
Gain	\$2,000	\$1,000	\$1,000
Cost of reinvestment			\$1,500
Net gain to stakeholders		\$1,000	\$-500

3. (Problem 2, pp. 530-531 in the textbook) The market for digital cameras is relatively new. Ajax Inc. produces what it regards as a high-quality digital camera. Knockoff Inc. produces what it regards as a low-quality digital camera. However, because the market is so new, reputations for quality have not yet developed, and consumers cannot tell the quality difference between an Ajax digital and a Knockoff digital just by looking at them.

If consumers knew the difference, they'd be willing to pay \$200 for a high-quality camera, and they'd be willing to pay \$100 for a low-quality camera. It costs Ajax \$85 to produce a high-quality camera, and it costs Knockoff \$55 to produce a low-quality camera.

A recent MBA hire at Ajax suggests that Ajax could differentiate its camera from Knockoff's by offering a full-coverage warranty (which would fully cover any defect in the camera at no cost to the customer). The MBA estimates that it would cost Ajax \$20 per year to offer such a warranty. The MBA also estimates that it would cost Knockoff \$40 per year should Knockoff attempt to copy Ajax's warranty strategy. Consumers will feel that the camera with the longest warranty is high-quality and that with the shortest warranty is low quality. The camera companies want to maximize the profit per camera.

What is Ajax's profit per camera in the digital camera market?

SOLUTION: If there were no informational asymmetries, then Ajax could sell its no high-quality camera for \$200 and earn a profit of \$115 per camera, and Knockoff would sell its low-quality camera for \$100 and earn a profit of \$45. However, since consumers cannot distinguish between high- and low-quality cameras, they are only willing to pay the low-quality camera price, which implies only \$15 profit for Ajax and \$45 profit for Knockoff. However, by offering a warranty, Ajax is able to credibly signal camera quality and therefore can expect to earn $\$200 - \$85 - \$20 = \95 per camera. There is no reason for Knockoff to offer a warranty, since all this will do is lower its profit from \$45 to \$5 per camera per year (the implicit assumption that I am making is that it is not possible for Knockoff to use a warranty strategy to falsely signal that its camera is of high quality when it is not).

This problem can also be viewed in a game theory context. The players are Ajax and Knockoff, the strategies are (warranty, no warranty), and the payoffs are given in the table below:

		Knockoff	
		Warranty	No Warranty
Ajax	Warranty	95,5	<u>95,45</u>
	No Warranty	15,5	15,45

The dominant strategy for Ajax is to provide a warranty, whereas the dominant strategy for Knockoff is to not provide a warranty.

4. (Problem 4, p. 531 in the textbook) Some people are good drivers and others bad drivers. The former have a 10% chance of crashing their cars and the latter have a 30% chance. All have a total wealth of \$400 but this will fall to \$100 if they crash their cars. In other words, each will lose \$300 if they crash. You are an insurance company who wishes to offer a pair of policies to all drivers. Each policy is designed to break even (zero profit) given the people that choose to buy that policy. The first policy has a premium of \$90 and covers all losses (i.e., will pay \$300 in the event of a crash. The second policy has a premium of \$5 and will pay \$50 in the event of a crash. Who will buy which policy? Will the insurance company, (a) make a profit, (b) break even or (c) lose money?

Each person has a utility function as follows

$$\text{Utility} = (\text{wealth})^{.5}$$

SOLUTION: We need to compute the expected utility that bad and good drivers obtain from these two contracts as well as being uninsured:

Good drivers:

- No insurance: $EU_G^{NI} = .1\sqrt{400 - 300} + .9\sqrt{400} = 19.$
- Full coverage: $EU_G^{FC} = \sqrt{400 - 90} = \sqrt{310} = 17.61.$
- Partial Coverage: $EU_G^{PC} = .1\sqrt{400 - 5 - 300 + 50} + .9\sqrt{400 - 5} = .1\sqrt{145} + .9\sqrt{395} = 19.09.$

Bad drivers:

- No insurance: $EU_B^{NI} = .3\sqrt{400 - 300} + .7\sqrt{400} = 17.$
- Full coverage: $EU_B^{FC} = \sqrt{400 - 90} = \sqrt{310} = 17.61.$
- Partial Coverage: $EU_B^{PC} = .3\sqrt{400 - 5 - 300 + 50} + .7\sqrt{400 - 5} = .3\sqrt{145} + .7\sqrt{395} = 17.52.$

Since $EU_G^{PC} > EU_G^{NI} > EU_G^{FC}$, the good drivers buy the partial coverage policy for \$5. Profits on this policy for the insurance company would be 0; the policyholders would pay \$5 for the policy, and the expected cost to the company would be \$5, since 10% of the time, good drivers crash their cars at a cost of \$50 to the insurer. Thus the insurance company would break even. Furthermore, since $EU_B^{FC} > EU_B^{PC} > EU_B^{NI}$, the bad drivers purchase the full coverage policy for \$90. Profits on this policy for the insurance company would be 0; the policyholders would pay \$90 for the policy, and the expected cost to the company would be \$90, since 30% of the time, bad drivers crash their cars at a cost of \$300 to the insurer. Thus the insurance company would also break even on the full coverage contract. Here, we have no adverse selection, since bad and good drivers self select into the policies which enable the insurer to break even.