BAYLOR UNIVERSITY
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ECO 5315 Chapter 13 Problem Set Solutions
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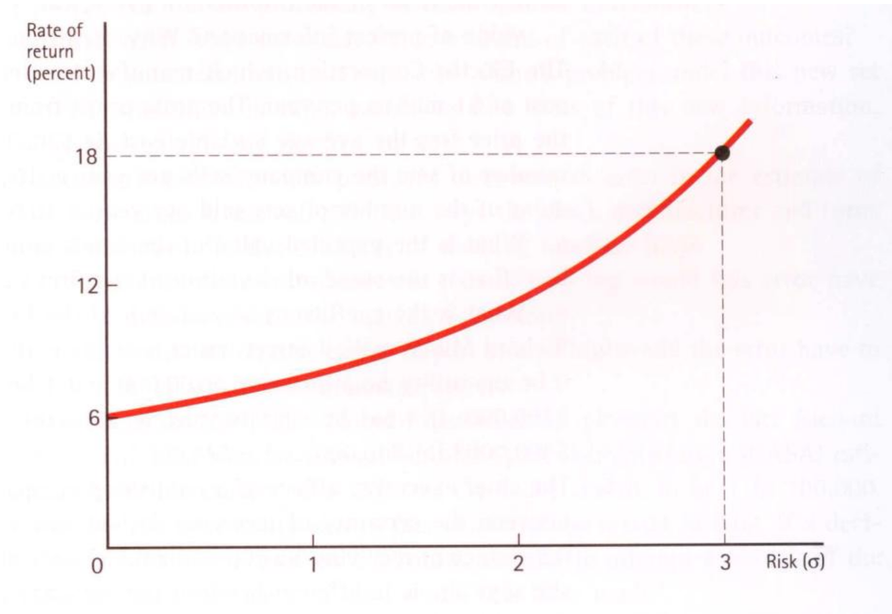
1. (Problem 2, pp. 456-457 in the textbook) William J. Bryan is the general manager of an electrical equipment plant. He must decide whether to install a number of assembly robots in his plant. This investment would be quite risky, since both management and the workforce have no real experience with the introduction or operation of such robots. His indifference curve between expected rate of return and risk is as shown in the figure.



A. If the riskiness ((σ) of this investment equals 3, what risk premium does he require?

<u>Solution:</u> The the risk premium is equal to the difference between the expected rate of return on a particular risky investment and that on a riskless investment. Thus, the risk premium is the extra return that is required for taking risk compared with not taking risk. In this figure, Mr. Bryan's required return for a riskless ($\sigma = 0$) is 6 percent, but for a risky investment with $\sigma = 3$, Mr. Bryan requires at least 18 percent. If this risky investment earned less than 18%, then Mr. Bryan's expected utility would be higher if he invested instead in the riskless investment. Therefore, a risk premium of 18 percent – 6 percent = 12 percent is required in order for Mr. Bryant to be indifferent between a 6 percent riskless ($\sigma = 0$) return and an 18 percent risky ($\sigma = 3$) return.

B. What is the riskless rate of return?

Solution: Here, it is 6 percent.

C. What is the risk-adjusted discount rate?

Solution: The risk-adjusted discount rate is that rate of return that Mr. Bryant would use in order to calculate the certainty-equivalent value of the expected cash flows from the risky project. Since the returns on this project have a standard deviation of 3, the risk-adjusted discount rate is 18 percent.

D. In calculating the present value of future profits from this investment, what interest rate should be used?

Solution: As is shown in equation (13.8) in the textbook, the appropriate interest rate to use is the risk-adjusted discount rate, which is 18 percent.

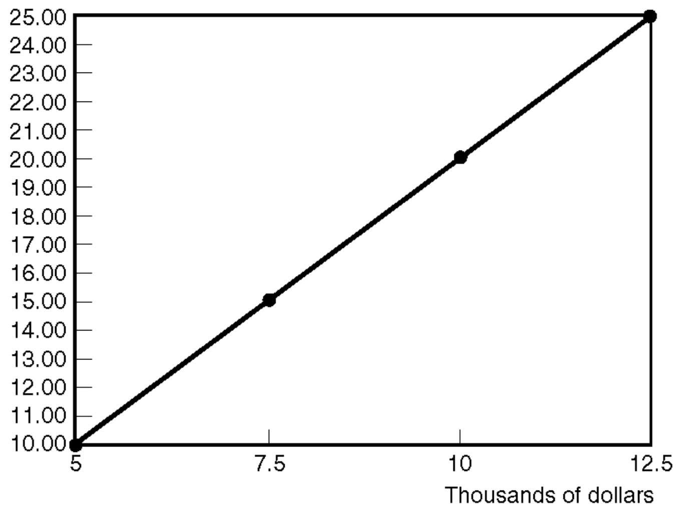
2. (Problem 6, page 459 in the textbook) The chief executive officer of a publishing company says she is indifferent between the certainty of receiving $7,500 and a gamble where there is a 0.5 chance of receiving $5,000 and a 0.5 chance of receiving $10,000. Also, she says she is indifferent between the certainty of receiving $10,000 and a gamble where there is a 0.5 chance of receiving $7,500 and a 0.5 chance of receiving $12,500.

Is this CEO a risk averter, a risk lover, or risk neutral? Explain.

Solution: This problem shows the same bet at two different levels of initial wealth. The bet involves equal probabilities of losing or winning $2,500. In the first scenario, the CEO has initial wealth of $7,500 and she is indifferent between this and a gamble involving equal probabilities of having either $5,000 or $10,000. In the second scenario, initial wealth is increased from $7,500 to $10,000 and the CEO is indifferent between having $10,000 for sure versus a gamble involving equal probabilities of having either $7,500 or $10,000.

Note that the expected values of betting versus not betting are the same for both wealth levels: $7,500 and $10,000. By not betting she does not take on any risk. However, by betting she does take on risk (of $\sigma = 2,500$), but she does not require a risk premium. Therefore, the CEO is risk neutral; it does not matter what her initial level of wealth is, because she is indifferent about risk.

Another way to show this is to simply draw a picture of her utility function. Irrespective of how you scale her utility, it is clear that the utility consequence of changes in wealth are linear in wealth; i.e., $U = a + bW$. For example, suppose $a = 0$ and $B = 2$. Then here is a picture of her utility for wealth of $5,000, $7,500, $10,000, and $12,500:



3. A worker whose utility function $U(W) = \sqrt{W}$ has received a job offer which pays $80,000 with a bonus. The bonus is equally likely to be $0, $10,000, $20,000, $30,000, $40,000, $50,000, or $60,000. Assume that initial wealth is $0.

   A. What is the expected value of this pay package?

   Solution: To compute the expected value of this pay package, we must solve the following equation: $E(W) = \sum_{s=1}^{n} p_s W_s$ = (1/7)(80,000) + (1/7)(90,000) + (1/7)(100,000) + (1/7)(110,000) + (1/7)(120,000) + (1/7)(130,000) + (1/7)(140,000) = $110,000.

   B. What is the certainty equivalent of this pay package?

   Solution: $E(U(W)) = \sum_{s=1}^{n} p_s U(W_s)$ = $(1/7)(80{,}000)^{.5} + (1/7)(90{,}000)^{.5} + (1/7)(100{,}000)^{.5} + (1/7)(110{,}000)^{.5} + (1/7)(120{,}000)^{.5} + (1/7)(130{,}000)^{.5} + (1/7)(140{,}000)$ = 330.27. Therefore, the certainty equivalent is $W_{CE} = E(U(W))^2 = 330.27^2$ = $109,075.82.

   C. What is the risk premium?

   Solution: The risk premium is the difference between the expected wealth and the certainty equivalent of wealth, i.e., $\lambda = E(W) - W_{CEQ}$ = $110,000 - $109,075.82 = $924.18.

4. Ken can choose one of two types of swings. He can try either to make contact or swing for the fences. If he tries to make contact, his distribution of outcomes will be (out, 0.65; single, 0.35). If he swings for the fences his distribution of outcomes will be (out, 0.75; double, 0.15; homerun 0.1). Assume his utility for an out is 0 and his utility for a homerun is 1.

A. Ken is indifferent between a single with certainty and with a lottery equal to (out, 0.5; homerun, 0.5). What is his utility for a single?

Solution:
U(single) = EU(out, 0.5; homerun, 0.5)
= 0.5(0) + 0.5(1)
= 0.5.

B. Ken is indifferent between a double with certainty and with a lottery equal to (single, 0.6; homerun, 0.4). What is his utility for a double?

Solution:
U(double) = EU(single, 0.6; homerun, 0.4)
= 0.6(0.5) + 0.4(1)
= 0.7.

C. If Ken maximizes expected utility, should he try to make contact or swing for the fences? Show your work.

Solution:
EU(contact) = 0.65(0) + 0.35(0.5) = 0.175
EU(fences) = 0.75(0) + 0.15(0.7) + 0.10(1) = 0.205

Since EU(fences) > EU(contact), Ken should swing for the fences

D. Ken is indifferent between a lottery equal to (single, 0.5; homerun, 0.5) and a lottery equal to (double, 0.8; triple, 0.2). What is his utility for a triple?

Solution:
EU(single, 0.5; homerun, 0.5) = EU(double, 0.8; triple, 0.2)
= 0.5(0.5) + 0.5(1) = 0.8(0.7)+0.2U(triple)
U(triple) = 0.95.

5. An individual with initial wealth of $400 has a 20% chance of getting in an accident. If he gets in an accident, he will lose $300, leaving him with $100; if he does not, he loses nothing. He maximizes expected utility, and his utility function is $U(W) = \ln W$.

A. What is the expected amount of money he will lose? What is his expected wealth?

Solution:
His expected loss is $E(L)$ = (.2)($300)+(.8)(0) =$60, and his expected wealth is $E(W)$ = (.2)($100)+(.8)($400) =$340.

B. What is his expected utility?

Solution:
His expected utility is $E(U(W)) = (.2)\ln 100 + (.8)\ln 400 = .921 + 4.793 = 5.714.$

C. What is his certainty equivalent wealth, i.e., the certain wealth level that gives him the same expected utility as his uncertain situation?

SOLUTION: In order to solve for the certainty equivalent of wealth, we set the utility of the certainty equivalent equal to the expected utility of wealth and perform the necessary algebra:

$$U(W_{CEQ}) = \ln W_{CEQ} = E(U(W)) = 5.714; \therefore W_{CEQ} = e^{5.714} = \$303.08.$$

D. What is the maximum amount he would pay for full insurance coverage, i.e., what is the maximum premium he would pay an insurance company to cover all of his losses?

SOLUTION: The maximum amount he would pay for full insurance is his initial wealth minus his certainty equivalent wealth: \$400 - \$303.08 = \$96.92. This would make him indifferent between being insured and not being insured.