BAYLOR UNIVERSITY
HANKAMER SCHOOL OF BUSINESS
DEPARTMENT OF ECONOMICS

ECO 5315 Chapter 7 and Chapter 10 Problem Set Solutions
Jim Garven
Fall 2009

Chapter 7 Problems

4. The Madison Corporation, a monopolist, receives a report from a consulting firm concluding that the demand function for its product is

$$Q = 78 - 1.1P + 2.3Y + 0.9A$$

where Q is the number of units sold, P is the price of its product (in dollars), Y is per capita income (in thousands of dollars), and A is the firm's advertising expenditure (in thousands of dollars). The firm's average variable cost function is

$$AVC = 42 - 8Q + 1.5Q^2$$

where AVC is average variable cost (in dollars).

   a. Can one determine the firm's marginal cost curve?

   SOLUTION: From the average variable cost function, we can obtain the total cost function; i.e., $TC = AVC(Q) = 42 - 8Q + 1.5Q^2 = 42Q - 8Q^2 + 1.5Q^3$. Therefore, $MC = dTC/dQ = 42 - 16Q + 4.5\ Q^2$.

   b. Can one determine the firm's marginal revenue curve?

   SOLUTION: Solving for P in the equation $Q = 78 - 1.1P + 2.3Y + 0.9A$, we find that $P = (78 - Q + 2.3Y + 0.9A)/1.1$. Therefore, $TR = PQ = [(78 - Q + 2.3Y + 0.9A)/1.1]Q$, and $MR = dTR/dQ = (78 - 2Q + 2.3Y + 0.9A)/1.1$. We need information on per capita income and on advertising expenditures to determine marginal revenue as a function of Q alone.

c. If per capita income is $4,000 and advertising expenditure is $200,000, can one determine the price and output where marginal revenue equals marginal cost? If so, what are they?

SOLUTION: Set $MR = MC$. $MR = [78 – 2Q + 2.3(4) + 0.9(200)]/1.1 = 242.9 – 1.82Q = 42 – 16Q + 4.5Q^2 = MC \Rightarrow 4.5Q^2 – 14.18Q – 200.9 = 0$.

Since the marginal cost equation is a quadratic (or second degree polynomial) equation of the form $ax^2 + bx + c = 0$, we can find Q by using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-14.18 + \sqrt{14.18^2 - 4(4.5)(-200.9)}}{2(4.5)} = Q = 5.29$.

Therefore, $P = (78 – Q + 2.3Y + 0.9A)/1.1 = (78 – 5.29 + 2.3(4) + 0.9(\$200))/1.1 = 238.1$.

6. If the Rhine Company ignores the possibility that other firms may enter its market, it should set a price of $10,000 for its product, which is a power tool. But, if it does so, other firms will begin to enter the market. During the next two years, it will earn $4 million per year, but in the following two years, it will earn $1 million per year. On the other hand, if it sets a price of $7,000, it will earn $2.5 million in each of the next four years, since no entrants will appear.

   a. If the interest rate is 10 percent, should the Rhine Company set a price of $7,000 or $10,000? Why? (Consider only the next four years.)

   SOLUTION: If a price of $10,000 is chosen, the present value of profits is $PV = \frac{\$4 \text{ million}}{1.10} + \frac{\$4 \text{ million}}{1.10^2} + \frac{\$1 \text{ million}}{1.10^3} + \frac{\$1 \text{ million}}{1.10^4} = \$8.376$ million, whereas if a price of $7,000 is chosen, the present value of profits is $PV = \sum_{t=1}^{4} \$2.5 \text{ million} / (1.1)^t = \$2.5 \text{ million} \times (1 - 1/(1.1)^4)/.1 = \$2.5$ million (3.1699) = $7.925 million. Therefore, a price of $10,000 should be chosen, since this price provides a higher present value of profits.

   b. If the interest rate is 8 percent, should the Rhine Company set a price of $7,000 or $10,000? Why? (Consider only the next four years.)

   SOLUTION: Since present values are inversely related to discount rates, the decision to set a price of $10,000 looks even better at 8 percent than it does at

10 percent. At an 8 percent interest rate, the present value of profits is $PV = \frac{\$4 \text{ million}}{1.08} + \frac{\$4 \text{ million}}{1.08^2} + \frac{\$1 \text{ million}}{1.08^3} + \frac{\$1 \text{ million}}{1.08^4} = \$8.662$ million if the price is set at \$10,000, whereas if a price of \$7,000 is chosen, then the present value of profits is $PV = \sum_{t=1}^{4} \$2.5 \text{ million} / (1.08)^t$

$= \$2.5 \text{ million} \times (1 - 1/(1.08)^4)/.08 = \$2.5 \text{ million } (3.3121) = \$8.28 \text{ million.}$

Note that if the interest rate were zero, then the two profit streams would have equal value. For any positive rate of interest, the \$10,000 price yields a higher present value because it generates the same undiscounted total cash flow and has more of it sooner than the \$7,000 price does.

c. The results in parts a and b pertain to only the next four years. How can the firm's managers extend the planning horizon?

SOLUTION: The firm would need to estimate profits for these additional years and include them in the present value calculations.

12. The Morrison Company produces tennis rackets, the marginal cost of a racket being \$20. Because there are many substitutes for the firm's rackets, the price elasticity of demand for its rackets equals about 22. In the relevant range of output, average variable cost is very close to marginal cost.

a. The president of the Morrison Company feels that cost-plus pricing is appropriate for his firm. He marks up average variable cost by 100 percent to set price. Comment on this procedure.

b. Because of heightened competition, the price elasticity of demand for the firm's rackets increases to 23. The president continues to use the same cost-plus pricing formula. Comment on its adequacy.

SOLUTION:

a. The profit-maximizing markup from marginal cost is $-1/(h + 1)$, which, if the elasticity is -2, equals 100 percent. If marginal cost equals average variable cost, then the 100 percent markup from average variable cost is profit maximizing.

b. If the elasticity changes to -3, then the profit-maximizing markup is 50 percent.

Chapter 10 Problems

2. The can industry is composed of two firms. Suppose that the demand curve for cans is

$$P = 100 - Q$$

where P is the price (in cents) of a can and Q is the quantity demanded (in millions per month) of cans. Suppose that the total cost function of each firm is

$$TC = 2 + 15q$$

where *TC* is total cost (in tens of thousands of dollars) per month and *q* is the quantity produced (in millions) per month by the Firm.

a. What are the price and output if the firms set price equal to marginal cost?

   SOLUTION: Since each firm has a constant marginal cost of $.15, the price must also equal $.15 for price to equal marginal cost. Since marginal cost equals price equals average variable cost in this case, each firm loses an amount equal to their fixed costs, $20,000.

b. What are the profit-maximizing price and output if the firms collude and act like a monopolist?

   SOLUTION: If they collude, they will produce where marginal revenue equals marginal cost. Marginal revenue is given by $MR = 100 - 2Q$. Setting marginal revenue equal to marginal cost, we get the joint profit maximizing combined output is $Q = 42.5$. Since the firms have constant marginal costs, only one firm should operate; thereby they avoid the fixed costs of the other firm. Their combined profits would be $\pi = 57.5(42.5) - [2 + 15(42.5)] = 1{,}804.25$, or $18,042,500. If they cannot avoid the fixed costs of one of the firms by shutting it down, their combined profits would be $18,022,500.

c. Do the firms make a higher combined profit if they collude than if they set price equal to marginal cost? If so, how much higher is their combined profit?

SOLUTION: Since they lose \$40,000 if they compete and earn \$18,022,500 if they collude, they earn \$18,062,500 more if they collude than if they compete.

4. James Pizzo is president of a firm that is the price leader in the industry; that is, it sets the price and the other firms sell all they want at that price. In other words, the other firms act as perfect competitors. The demand curve for the industry's product is $P = 300 - Q$, where P is the price of the product and Q is the total quantity demanded. The total amount supplied by the other firms is equal to Q_r, where Q_r = $49P$. (P is measured in dollars per barrel; Q, Q_r, and Q_b are measured in millions of barrels per week.)

    a. If Pizzo's firm's marginal cost curve is $2.96Q_b$ where Q_b is the output of his firm, at what output level should he operate to maximize profit?

    SOLUTION: The residual demand faced by Pizzo is $Q - Q_r = 300 - P - 49P = 300 - 50P$, which can be written as $P = 6 - 0.02Q_b$. Setting his marginal revenue equal to his marginal cost, $6 - 0.04Q_b = 2.96Q_b \Rightarrow Q_b = 2$, Pizzo determines that the profit maximizing quantity is 2.

    b. What price should he charge?

    SOLUTION: $P = 6 - 0.02(2) = \$5.96$.

    c. How much does the industry as a whole produce at this price?

    SOLUTION: $Q = 300 - P = 294.04$, $Q_r = 49(596) = 292.04$.

    d. Is Pizzo's firm the dominant firm in the industry?

    SOLUTION: No. The elasticity of Pizzo's residual demand at his chosen level of output is nearly 150. Notice that his marginal cost is \$5.92 and yet his market power only allows him to charge a price of \$5.96. He produces less than 1 percent of the industry's output.

12. Steve Win has purchased land (an old landfill-from rags to riches) from the city of Atlantic City in the Marina section of the city. There are stories of a new casino building boom in Atlantic City (MGeeM is also talking about entering and Gump is opening his fourth casino). Some talk is circulating that Win will subdivide his new land purchase and perhaps three casinos will be built on the site.

    Suppose that Win subdivides his land into two parcels. He builds on one site and sells the other to another gambling entrepreneur. Win estimates that the demand for gambling in the Marina area of Atlantic City (*After* accounting for the presence of two existing casinos in the Marina and adjusting for the rest of the casinos in Atlantic City) to be

    $$P = 750 - 5Q$$

    where P is the price associated with gambling and Q is the quantity of gambling (think of P as the average amount that a typical patron will net the casino, an amount paid for the entertainment of gambling, and Q as the number of gamblers).

    Win, of course, does not sell the other parcel until his casino is built (or is significantly far along); thus, he has a first-mover advantage.

    Win's total cost (TC_w) of producing gambling is

    $$TC_W = 20 + 40Q_W + 15.5Q_W^2$$

    where Q_W is the number of gamblers in Win's casino and the total cost (TC_R) of producing gambling for Win's rival is

    $$TC_R = 10 + 50Q_R + 20Q_R^2$$

    where Q_R is the number of gamblers in the rival's casino and

    $$Q_W + Q_R = Q$$

Would Atlantic City have done better to sell the land as two separate parcels rather than as a single parcel to Win (given that Win was going to subdivide, Win and his rival could not collude, and Win did not have the ability to produce as a monopolist)? You may assume that Win and his rival could have been Cournot duopolists. If Atlantic City could do better, show why and by how much? Carry all calculations to the thousandths decimal point.

SOLUTION: As I pointed out in my email/blog entry from November 2, we assume that the objective for Atlantic City is to maximize the quantity of gambling. The answer to this question involves a comparison of the outcomes that occur under the Cournot equilibrium compared with the Stackelberg equilibrium (where Win is the leader and the rival is the follower). The two-parcel setup is suggested in order to motivate a Cournot solution (where Win and Win's rival make simultaneous decisions), whereas the constrained one-parcel setup is presented in order to motivate a Stackelberg solution.

Therefore, suppose that Win and Win's rival are a Cournot duopoly. Since $P = 750 - 5Q \Rightarrow P = 750 - 5Q_W - 5Q_R$. Therefore, total revenue for Win (TR_W) is:

$TR_W = (750 - 5Q_W - 5Q_R)Q_W = 750Q_W - 5Q_W^2 - 5Q_WQ_R$; thus
$MR_W = 750 - 10Q_W - 5Q_R = 40 + 31Q_W = MC_W \Rightarrow 710 - 41Q_W - 5Q_R = 0 \Rightarrow$
$Q_W = 17.317 - .122Q_R$, which is Win's reaction function.

Similarly, total revenue for Win's (TR_R) is:

$TR_R = (750 - 5Q_W - 5Q_R)Q_R = 750Q_R - 5Q_R^2 - 5Q_WQ_R$; thus
$MR_W = 750 - 10Q_R - 5Q_W = 50 + 40Q_R = MC_R \Rightarrow 700 - 50Q_R - 5Q_W = 0 \Rightarrow$
$Q_R = 14 - .1Q_W$, which is Win's rival's reaction function.

Next, substitute the right-hand side of Win's rival's reaction function into Win's reaction function and solve for Q_W, Q_R, Q, P, π_W, and π_R:

$Q_W = 17.317 - .122(14 - .1Q_W) \Rightarrow Q_W = 15.8 \Rightarrow Q_R = 14 - .1(15.8) = 12.42$. Thus,
$Q = Q_W + Q_R = 15.8 + 14.22 = 28.22 \Rightarrow P = 750 - 5(28.22) = \$608.90$. Furthermore,
$\pi_W = 608.90(15.8) - 20 - 40(15.8) - 15.5(15.8^2) = \$5{,}099.20$, and
$\pi_R = 608.90(12.42) - 10 - 50(12.42) - 20(12.42^2) = \$3{,}846.91$.

Now, suppose that Win is the leader and Win's rival is the follower. We know that $TR_W = 750Q_W - 5Q_W^2 - 5Q_W Q_R$; substituting the right-hand side of Win's rival's reaction function, we obtain $TR_W = 750Q_W - 5Q_W^2 - 5Q_W(14 - .1Q_W) =$ $680Q_W - 4.5Q_W^2 \Rightarrow MR_W = 680 - 9Q_W = 40 + 31Q_W = MC_W \Rightarrow Q_W = 16$. Therefore, $Q_R = 14 - .1(16) = 12.4 \Rightarrow Q = Q_W + Q_R = 16 + 12.4 = 28.4 \Rightarrow P = 750 - 5(28.4) = \$608$. Furthermore, $\pi_W = 608(16) - 20 - 40(16) - 15.5(16^2) = \$5,100$, and $\pi_R = 608(12.4) - 10 - 50(12.4) - 20(12.4^2) = \$3,834$.

In summary, the price-quantity pair in the Cournot equilibrium is ($608.90, 28.22), whereas the price-quantity pair in the Stackelberg equilibrium is ($608, 28.4). If Atlantic City's objective is to structure the deal so as to maximize the quantity of gambling, then it should sell the land as single parcel, with the *quid pro quo* that Win subdivide the parcel and sell it to a rival for the same price that would have obtained had the city sold the land as two separate parcels (to Win and Win's rival) in the first place.